Oscillation Problems III

1. A simple pendulum has a period of 1.5 seconds on the earth. If the same pendulum on another planet has a period of 3 seconds, what is the acceleration due to gravity on that other planet?

$T_1 = 1.5\ s$
$g_1 = 10\ m/s^2$
$T_2 = 3\ s$
$g_2 = ?$

$T = 2\pi\sqrt{\frac{L}{g}}$

$1.5 = 2\pi\sqrt{\frac{L}{10}}$

$\frac{(1.5)^2(10)}{4\pi^2} = L$

$L = 0.57\ m$

$T = 2\pi\sqrt{\frac{L}{g}}$

$T^2 = 4\pi^2\frac{L}{g}$

$g = \frac{4\pi^2 L}{T^2} = \frac{4\pi^2(0.57)}{(3)^2}$

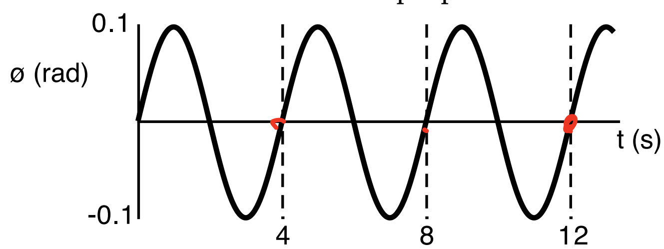
$\boxed{g = 2.5\ m/s^2}$

OR $\frac{T_1}{T_2} = \frac{2\pi\sqrt{\frac{L}{g_1}}}{2\pi\sqrt{\frac{L}{g_2}}}$

$\frac{T_1}{T_2} = \sqrt{\frac{g_2}{g_1}}$

$g_2 = \left(\frac{T_1}{T_2}\right)^2 g_1 = \left(\frac{1.5}{3}\right)^2 10$
$= 2.5\ m/s^2$

2. The angular position as a function of time for a simple pendulum is shown below.



a. What is the length of the pendulum?

3 cycles in 12 s

$\therefore T = \frac{12}{3} = 4\ s$

$A = 0.1\ rad$

$T = 2\pi\sqrt{\frac{L}{g}}$

$4 = 2\pi\sqrt{\frac{L}{10}}$

$\boxed{L = 4.05\ m}$

b. What is the maximum linear speed of the mass at the end of the simple pendulum?

$T = \frac{2\pi}{\omega}$

$\omega = \frac{2\pi}{4} = \frac{\pi}{2}\ rad/s$

Careful!
ω is angular frequency – it is connected to the period of the motion, but it is NOT the angular speed of pendulum!

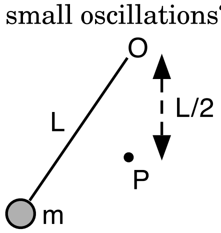
$\dot{\theta}_{max} = A\omega$
$= (0.1)(\pi/2)$

$\dot{\theta}_{max} = 0.157\ rad/s$

That is max angular speed.

$v_{max} = L\dot{\theta}_{max}$
$= (4.05)(0.157)$
$\boxed{= 0.637\ m/s}$

3. A simple pendulum of mass m and length L is hanging from a point "O." Directly underneath "O" is a pin "P" that is fixed in place. When the pendulum is released, the pin P becomes the new oscillation axis for that half of the motion. P is L/2 beneath O. What is the resulting period of small oscillations?



The left half of the motion is length L
The right half of the motion is length L/2

$\therefore \frac{1}{2}T_1 + \frac{1}{2}T_2 = \text{period}$

$\frac{1}{2}\left(2\pi\sqrt{\frac{L}{g}}\right) + \frac{1}{2}\left(2\pi\sqrt{\frac{L}{2g}}\right)$

$= \pi\sqrt{\frac{L}{g}} + \pi\sqrt{\frac{L}{2g}} = \pi\sqrt{\frac{L}{g}} + \frac{\pi}{\sqrt{2}}\sqrt{\frac{L}{g}} = \boxed{\pi\left(1+\frac{1}{\sqrt{2}}\right)\sqrt{\frac{L}{g}}}$

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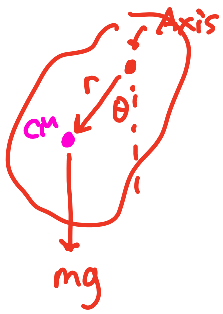
4. A physical pendulum is any body that is hung from a point (not its center of mass) and set oscillating back and forth. Calling the mass of the body "m" and the distance between the axis and the center of mass "r" and its moment of inertia about that axis "I", what is the period of small oscillation for a physical pendulum?



$\Sigma\tau = I\alpha$

$-rmg\sin\theta = I\ddot{\theta}$

"–" indicates torques are pushing object to hang straight down.

$\ddot{\theta} = \dfrac{-rmg\sin\theta}{I}$

$\ddot{\theta} \approx -\dfrac{rmg}{I}\theta$ b/c $\sin\theta \approx \theta$

SHM! & $\omega^2 = \dfrac{rmg}{I}$

So $T = \dfrac{2\pi}{\omega}$

$$\boxed{T = 2\pi\sqrt{\frac{I}{rmg}}}$$

5. What is the period of small oscillation for a thin rod of mass M and length L that is oscillating about one of its end points?

$$T = 2\pi\sqrt{\frac{I}{rmg}}$$

$$= 2\pi\sqrt{\frac{\frac{1}{3}ML^2}{\frac{L}{2}Mg}}$$

$$= 2\pi\sqrt{\frac{2}{3}\frac{L}{g}}$$

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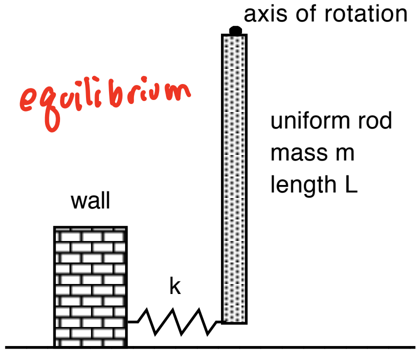
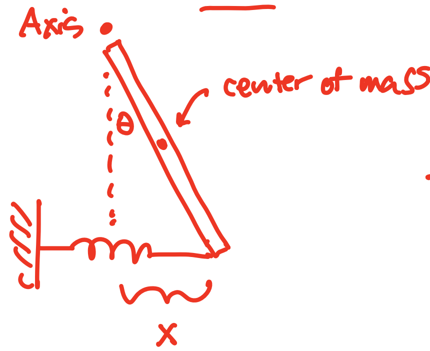
6. A thin rod of mass 400 grams and length 75 cm is suspended from one of its ends. At its other end is a small spring (k = 125 N/m) attached horizontally to a wall. The system is in equilibirum when it is hanging vertically. What is the period of small oscillation?

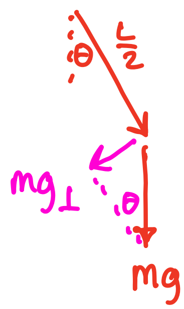
equilibrium





There are 2 forces:
one from gravity
&
one from the spring

gravity:

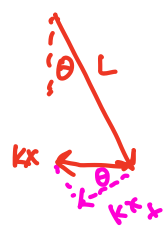


$mg_\perp = mg\sin\theta$

$\therefore\ \tau_1 = \left(\frac{L}{2}\right) mg\sin\theta$

spring: There are a few ways to do this, but it is always $\tau = \vec{r} \times \vec{F}$ where $r = L$ & F is kx (the force in the spring)

(A) From diagram



$kx_\perp = kx\cos\theta$

Also $x = L\sin\theta$

So $\tau_2 = L\,kx\cos\theta$

$= L\,k(L\sin\theta)\cos\theta$

$= kL^2\sin\theta\cos\theta$

For small angles, $\cos\theta \approx 1$, so

$\tau_2 \approx kL^2\sin\theta$

Answers:

1) 2.5 m/s² *2. a) 4.05 m* *b) 0.64 m/s* *3)* $\pi(1+\sqrt{2})\sqrt{\frac{L}{2g}}$ *4)* $2\pi\sqrt{\frac{I}{rmg}}$ *5)* $2\pi\sqrt{\frac{2L}{3g}}$

6) 0.203 s

(B) Could also argue that, for small angles, the spring is basically $\perp$ to the rod, so

$$\tau_2 \simeq L(kx)$$

and again we say $x = L\sin\theta$

So $\underline{\tau_2 = kL^2 \sin\theta}$

(C) Could also argue, since small angles, the spring is $\perp$ to the rod AND the stretch in the spring is basically an arc length, so $x = L\theta$

So $\underline{\tau_2 = kL^2 \theta}$

Note: in (A) & (B) we will end up saying $\sin\theta \simeq \theta$, so in the end, they are all the same...

Now, let's do it for real:

$$\Sigma \tau = I\alpha$$

$$\tau_1 + \tau_2 = \frac{1}{3} mL^2 \ddot{\theta}$$

$$\frac{1}{3} mL^2 \ddot{\theta} = -kL^2 \sin\theta - \left(\frac{L}{2}\right) mg \sin\theta$$

$$\ddot{\theta} = -\frac{3k}{m} \sin\theta - \frac{3g}{2L} \sin\theta$$

$$\ddot{\theta} \simeq -\frac{3k}{m}\theta - \frac{3g}{2L}\theta = -\left(\frac{3k}{m} + \frac{3g}{2L}\right)\theta$$

Hey! That's SHM, so $\omega^2 = \frac{3k}{m} + \frac{3g}{2L}$

$$\therefore \quad \boxed{T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\underbrace{\frac{3k}{m}}_{\text{a spring-like term}} + \underbrace{\frac{3g}{2L}}_{\text{a pendulum-like term}}}}}$$

So plugging in the #'s

$$T = \frac{2\pi}{\sqrt{\frac{3(125)}{(.4)} + \frac{3(10)}{2(.75)}}}$$

$$\boxed{T = 0.203 \text{ s}}$$